PRIVATE EQUITY
MATHEMATICS

Edited by Professor Oliver Gottschalg

Private equity as part of your portfolio

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Introduction

It is generally agreed that portfolios go through the classic life cycle of construct, nurture and harvest,[1] a process about which there is abundant research and views on managing various types of investment portfolios. Most of the research expounds upon the general principals articulated by Harry Markowitz in the *Journal of Finance* (1952) that serves as the foundation of the Modern Portfolio Theory (MPT). The technical research assumes that a portfolio is comprised of assets that are among other things fungible, transparent, readily quoted and easily transferable. These elements contribute towards understanding the risk-reward trade-offs between the investment choices thereby allowing the portfolio manager to craft an appropriate portfolio given their individual utility function.

The private equity (PE) market raises very unique challenges, including:

- *Construct phase* – lack of unitised/clean data, non-uniform access with generally large minimums, cash flow uncertainty and multi-year commitments, qualitative aspects (e.g., talent, relationships etc.) and other such elements.
- *Nurture phase* – lack of ability to actively manage or influence could vary from being completely passive for limited partners (LP) to being active for general partners (GP). However, in post-portfolio construction (or target acquisition) even the most active GPs can do little other than continue to be active on the individual portfolio companies themselves.
- *Harvest phase* – lack of multiple or defined exit options imply realisations could be sub-optimal or span many years. Continuing development of the secondary markets, structured products and listed PE funds notwithstanding, exit options are quite limited thereby making the asset class illiquid.

Further, the PE industry as a whole is not known to maintain very robust datasets, which includes issues such as lack of depth, information lag, lack of true price discovery, as well as selection and self-reporting biases. Reported returns are not normally distributed and also capital weighted thereby making uniform, unitised allocation analysis very difficult. It can also be generally agreed that possibly the most important aspect of PE portfolio management is the upfront selection whether it be the investment with a GP or the investment that a GP makes. Therefore, given the uniqueness of the PE markets, data issues and overlay of multiple non-quantifi-

able elements, PE portfolio management is as much an art as a science. However, given various non-technical elements, even if it is not possible to clearly articulate the exact methods of portfolio management, it may be possible to identify some general parameters, principals and metrics (herein, collectively called the 'private equity tools' (PET)). The potential application of the PETs in managing PE portfolios is unique to the type of participant, where:

- GPs focus on industry sub-sectors[2] (e.g., IT, industrial, etc.)
- Fund of funds focus on various types of GPs (e.g., buyout, secondary, etc.)
- Investors focus on types of investments (e.g., PE, public equity, bonds, etc.)

This chapter begins by presenting select PETs and then performs sample analyses from the perspective of each PE participant (i.e., GP, fund of funds and investors). The overall intent of the chapter is to demonstrate methods of estimating the PETs as well as highlighting PE participant-specific illustrations and presentation styles. At the onset, it is also equally important to remind the reader of the numerous concerns highlighted previously, therefore the results should be used with extreme caution and more so as relative anchor points used with some degrees of freedom.

Private equity tools (PETs)

As with all market practitioners, PE participants have their own preferences about the tools they use for portfolio management. However, although the tools, exact formulae and their utility may vary across the practitioners the analysis itself can be grouped into three general categories: (a) return-related, (b) risk-related, and (c) portfolio level. This section presents select PETs and their estimation formulae for each of the three general categories.

Return-related

Expected return is the mathematical expectation of return from a single or portfolio of holdings generally based on the expected probability of each return. This is expressed as follows:

$$E(R) = \sum_{i=1}^{n} P_i R_i$$

where
R_i is the possible return and P_i is the possibility of that outcome.

In quantifying the expected return, it is also important to establish the parameters around the expected return or whether it is: (a) relative or absolute, and (b) cash-on-cash or in percentages e.g., a 2x multiple return is 41 percent IRR if cash is returned in year two versus 10 percent IRR if cash is returned in year seven.

Mean return is the arithmetic average of the return. *Weighted average mean return* would include an additional set of information along with the return for the holding e.g., assets, number, capital, etc.

$$\bar{x} = \sum_{i=1}^{n} w_i x_i$$

where

$$\sum_{i=1}^{n} w_i = 1$$

Quartile is the measure of the relative ranking of the holding (e.g., return). The *K*th quartile of population *X* can be defined as the value '*x*' such that:

$$P(X \leq x) \leq p \text{ and } P(X \geq x) \geq 1 - p$$
$$\text{where } p = \frac{k}{4}$$

Risk-related

Standard deviation is the measure of deviation of values from their mean. It can be used as a measure of risk appetite or the tolerance of volatility on a single and/or a portfolio of investments. The more recent accounting pronouncements (e.g., US GAAP FAS 157) to mark holdings to market would create further intra-period valuation fluctuations.

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

Variance is the measure of dispersion around a value (e.g., expected return).

$$\text{Var}(x) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

Semi-deviation is the measure of volatility of values that occur below a target return.

$$\sqrt{\sum_{i=1}^{n} \frac{[\max(0, T - x_i)]^2}{n}}$$

where
T is the target return, and for this chapter the target return is assumed to be 8 percent.

Loss probability is the measure of the probability of loss in the portfolio. Some loss probability (SLP) is the ratio of results whose return is negative to the total number of returns. Assuming M possible results from a simulation, and S of them have negative returns, then the SLP is 100 (S/M) percent. Similarly, total loss probability (TLP) is the ratio of numbers of negative one return to the total number.

Portfolio level

Barbell strategy is portfolio composition with extreme holdings. For example, assuming an allocation to two positions then one position each in lower risk-return and higher risk-return assets (e.g., equal allocation to 1.5x and 2.5x assets with expected value at 2x).

Bullet strategy or *concentrated strategy* refers to portfolio composition with mid-value or target holdings. For example, assuming an allocation to two positions then both positions in mid risk-return assets (e.g., equal allocation to 2x and 2x assets with expected value at 2x). Further, tolerance thresholds can drive the level of skewness in the portfolio.

Correlation is the measure of relationship between two values. Estimates range from -1 (exact opposite relationship) to +1 (exact relationship).

$$\rho_{xy} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{(n-1)\sigma_x \sigma_y}$$

where
$\bar{x}, \bar{y}$ are the mean of x_i, y_i , and σ_x, σ_y are the standard deviation of x_i, y_i.

Intra-portfolio correlation is the measure of diversification within a portfolio or the degree of correlation of holdings within the portfolio.

$$Q = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j P_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j}$$

where
X_i, X_j are the fraction invested in asset i, j, and P_{ij} is the correlation between asset i and j.

Skewness is the measure of symmetry of the holdings within the portfolio. Positive means elongated right tail and negative means elongated left tail.

$$r_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{\dfrac{n}{\sigma_x^3}}$$

Kurtosis is the measure of the peakedness of the holdings within the portfolio. High kurtosis implies distinct peak with rapid decline. Low kurtosis implies flat top with slow decline.

$$r_2 = \frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})^4}{\dfrac{n}{\sigma_x^4}}$$

Random variable is the sequence of outcomes that do not have a defined pattern. There are many methods of generating random variables and some have been automated by third-party vendors like Matlab [3] (financial applications software from The MathWorks). One is the method suggested by D.H. Lehmer, where the generators have three integer parameters: a, c and m, and an initial value x_0, called the seed. A sequence of integers is defined by:

$$x_{k+1} = ax_k + c \bmod m$$

The operation 'mode m' means taking the reminder after division by m.

Sharpe ratio is the measure of the derived risk premium per unit of risk. Higher Sharpe ratios imply greater relative risk-adjusted value.

$$r_3 = \frac{E(R - R_f)}{\sqrt{\text{Var}(R - R_f)}}$$

where
R_f is the risk-free rate. For this chapter the risk-free rate is assumed to be 3 percent.

Sortino ratio is the measure of the risk-adjusted return of the portfolio. As a modification of the Sharpe ratio, it measures the derived risk premium over a target return.

$$r_4 = \frac{E(r) - T}{\text{Semi-deviation}}$$

where
T is the target rate. For this chapter the target return is assumed to be 8 percent.

The subsequent sections estimate these PETs using defined sources of datasets for each of the PE participants. The dataset can be analysed on a standalone or on a simulated basis. Standalone analysis would mean that the dataset itself is used for estimating these PETs. Simulation would mean that the dataset is used as a reference point by a random process that consequently builds a distribution of results that is used for estimating the PETs. For this chapter, the Matlab software has been used for simulating the expected-return distributions from the sample datasets over

[200,000] iterations. Thereafter the expected-return distributions were used to calculate the PETs based on the aforementioned formulae. For simplicity purposes, it was assumed that the datasets are perfectly unitised with the ability to make equal allocations on each iteration.

General partner perspective

The previous section identified select PETs that can be used for portfolio management while this section examines their application from a GP's perspective. GPs use a top-down (driven by macroeconomic factors or themes), bottom-up (opportunistic development) or a combination approach in managing their PE portfolios. Overall, portfolio management techniques will anchor around the composition of the GP's talent pool, which includes: (a) investment access, (b) ability to add value, (c) amount of disposable capital, (d) breadth and depth of team, and (e) incremental capital need/access. For example, GPs managing larger funds may devote resources to develop sector specific teams/themes but, given their fund size they generally do not participate in smaller deals. On the other hand, GPs managing smaller funds can remain specialists but they may trade-off on the size of the deal or on control. So, portfolios mean different things to different GPs, where there can be various quantifiable as well as non-quantifiable reasons for allocating capital across and within the different companies or sectors in the portfolio.

The PETs can help GPs perform quantitative sector analysis. In conducting this analysis, given the previously articulated data issues, the private market dataset was supplemented with the public market sector data as a proxy. For this section, the two datasets have (a) only included sectors where both datasets had more robust information, and (b) been organised into the following four sector groups to be better aligned.

- *Industrial* includes energy-related, chemicals, construction materials, metals, packaging materials, capital goods, including aerospace and defence, construction, engineering & building products and industrial machinery.
- *Consumer* includes food, beverages, personal products, supermarkets, retail, apparel, hotels & restaurants, media production and services, durable and non-durable household goods.
- *Healthcare* includes providers of healthcare products, services & operators of healthcare facilities, R&D, marketing & production of pharmaceuticals and biotech.
- *IT* includes software, hardware, database management, IT consulting & services, Internet, telecom (fixed and wireless), semiconductors.

It is possible to use different datasets and time periods to organise this data into smaller or larger groups as well as to include other sectors or groupings. For these illustrations, the information included the one-year, three-year, five-year, ten-year, 15-year and 20-year sector returns that were sourced private and public market returns.

- Private market returns of US pooled industry net IRRs from Thomson Financial (beginning in 1991) which include the information housed by Thomson Financial on all private equity transactions (e.g., buyouts, mezzanine, and venture capital) as sourced in July 2008.
- Public market returns of US industry indices from MSCI (beginning in 1996). The three-year, five-year, and ten-year cumulative industry returns provided by MSCI were annualised.

Return-related

This generally varies across sectors, time periods or horizons and economic cycles. It is not unusual for certain sectors to have significant relative over-performance over certain periods of time (e.g., IT in 1998 or 1999). Figures 8.1 and 8.2 show the one-year, three-year, five-year and ten-year historical returns across the four pre-defined sectors.

Figure 8.1: **Historical returns across pre-defined sectors – private market**

Figure 8.2: **Historical returns across pre-defined sectors – public market**
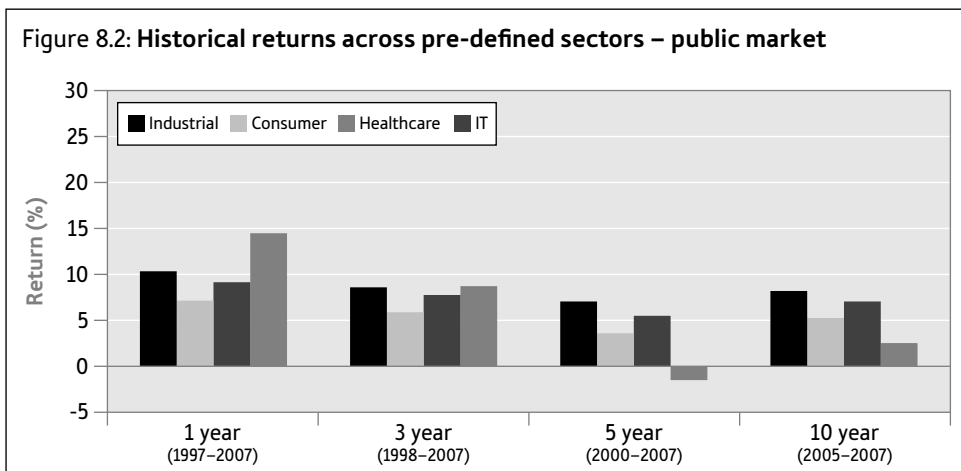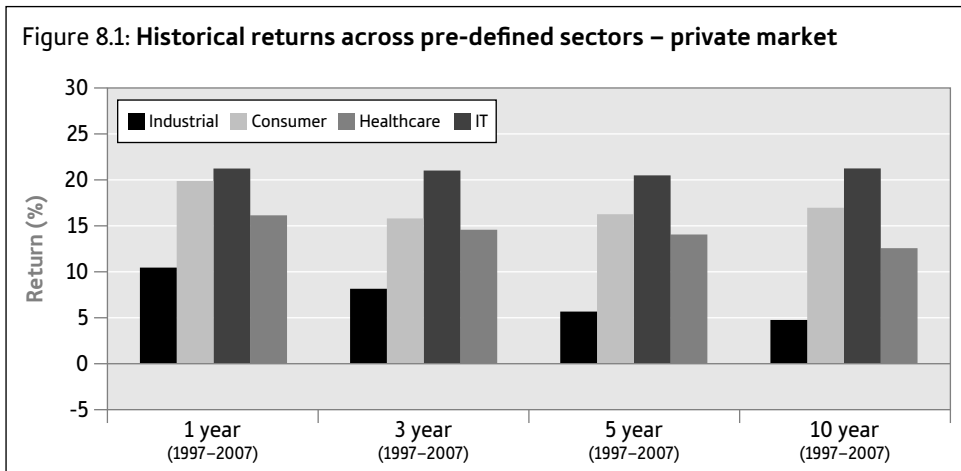
Figure 8.3: **Quartile returns, 1 year (1997–2007) – private**
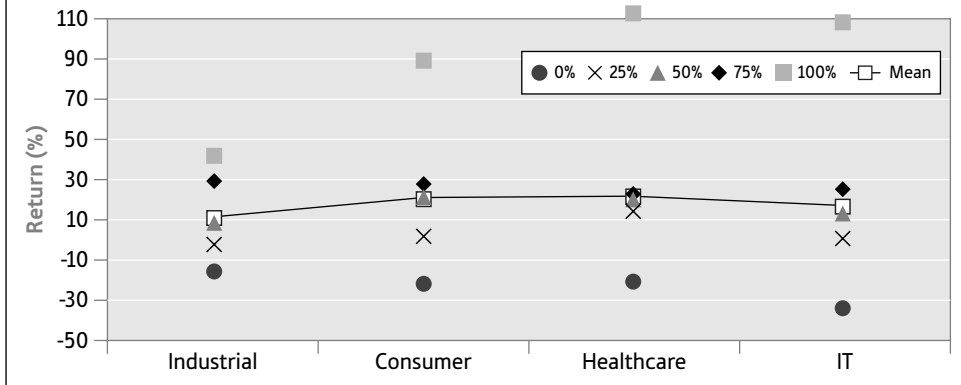
Figure 8.4: **Quartile returns, 1 year (1997–2007) – public**
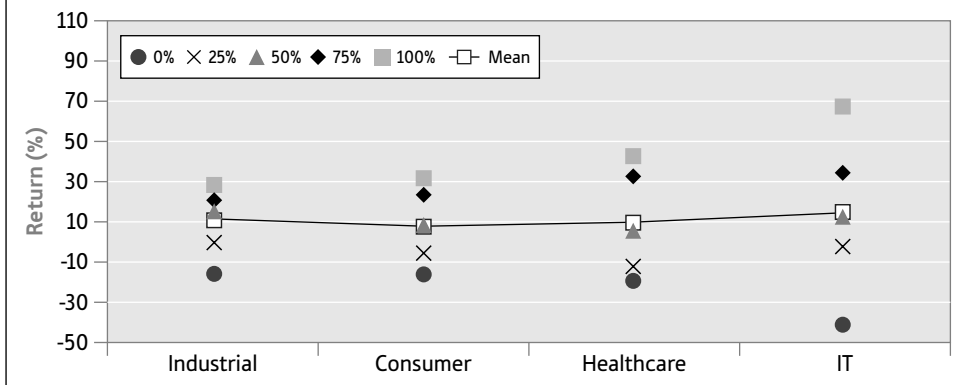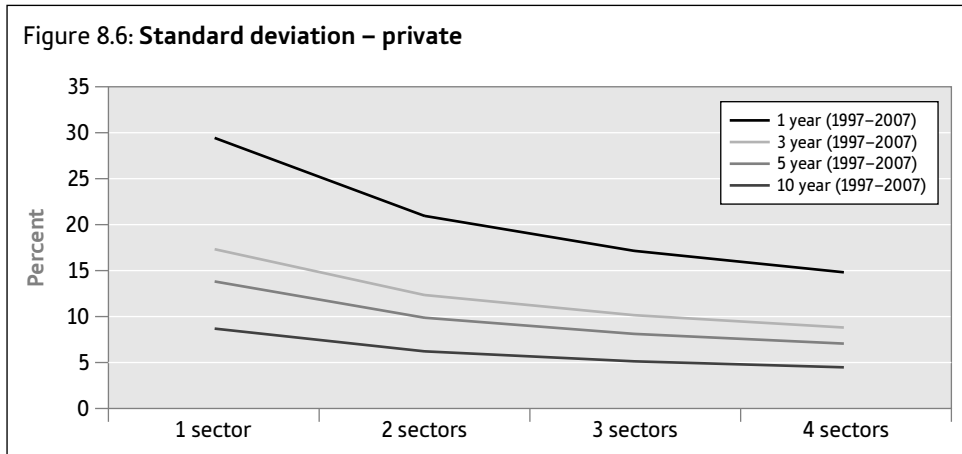

Figure 8.5: **Expected return distribution, 1 year (1997–2007) – private**

Figure 8.6: Standard deviation – private

Percent

Legend:
- 1 year (1997–2007)
- 3 year (1997–2007)
- 5 year (1997–2007)
- 10 year (1997–2007)

X-axis: 1 sector, 2 sectors, 3 sectors, 4 sectors

Figure 8.7: Standard deviation – public

Percent

Legend:
- 1 year (1997–2007)
- 3 year (1998–2007)
- 5 year (2000–2007)
- 10 year (2005–2007)

X-axis: 1 sector, 2 sectors, 3 sectors, 4 sectors
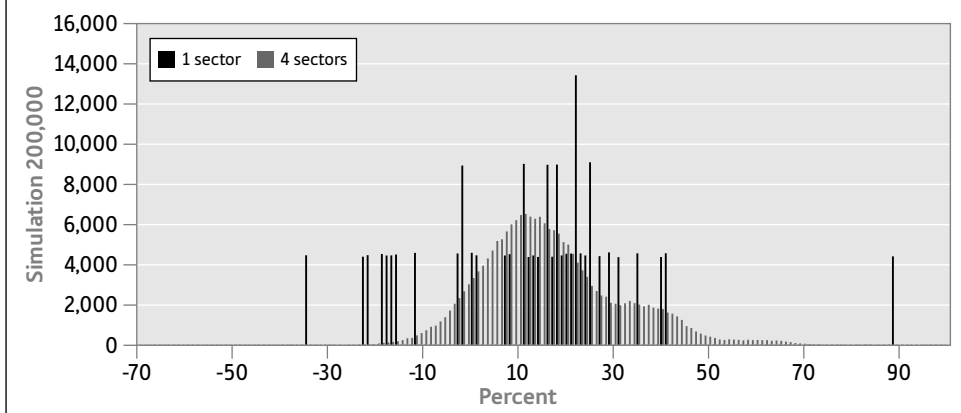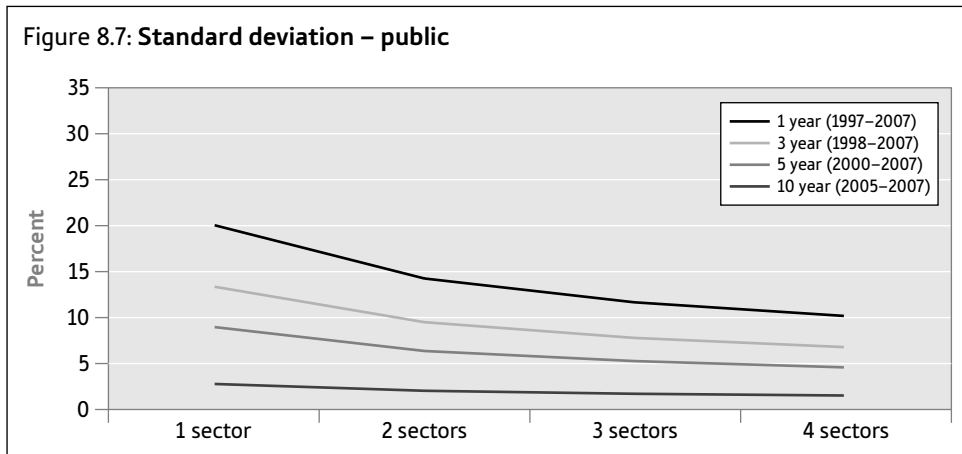
Figures 8.3 and 8.4 show that there is significant dispersion in the returns and that top-quartile returns exhibit high over-performance when compared to the mean returns.

Figure 8.5 shows that the expected return distribution takes on a more familiar or defined profile as more sectors are included in the portfolio. The expected return distributions can be constructed using: (a) random sector selection, or (b) anchoring the first sector and then introducing one more sector at a time. The results of the second approach should gradually approach the results of the first approach as more sectors are included. The deviation within the approaches will generally depend on the correlation between the sectors.

Risk-related

Standard deviation generally reduces as more sectors are included in the portfolio. Figures 8.6 and 8.7 show that this is true for both private and public portfolios.

Table 8.1: Correlation between the one-year returns of the PE sectors

	Industrial	Consumer	Healthcare	IT
Industrial	1.00	0.57	0.48	0.53
Consumer	0.57	1.00	0.92	0.94
Healthcare	0.48	0.92	1.00	0.96
IT	0.53	0.94	0.96	1.00

Table 8.2: Correlation between the returns of the various holding periods for the industrial sector

	1 year	3 year	5 year	10 year
1 year	1.00	0.78	0.71	0.48
3 year	0.78	1.00	0.88	0.70
5 year	0.71	0.88	1.00	0.80
10 year	0.48	0.70	0.80	1.00

However, diversification benefits are not limitless and it is important to be cognizant of the marginal impact of standard deviation versus return so as not to over diversify and return to mean.

Portfolio level

Correlation varies across sectors, time periods or horizons as well as vintages. Table 8.1 shows the correlation between the one-year returns of the PE sectors and Table 8.2 shows the correlation between the returns of the various holding periods for the industrial sector.

Since public and private datasets have been used, Table 8.3 shows the correlation between the sectors within each group. Table 8.4 measures the intra-portfolio correlation and shows that the individual sectors provide some level of diversification to each other.

Other metrics also vary across sectors, time periods or horizon as well as vintages. Tables 8.5 and 8.6 show that both data-series have a positive skew and some Kurtosis. Further, the Sharpe and Sortino ratios become better as more sectors are added.

Fund of fund portfolio perspective

The previous sections examined select PETs, their estimation formulae as well as their application for GPs. This section looks at PET application for fund of funds or for that matter investors looking at developing PE programmes (e.g., PE allocations

Table 8.3: Correlation between sectors within each group

	Industrial – public	Consumer (a) – public	Healthcare – public	IT – public
Industrial – private	0.77	0.21	-0.01	0.19
Consumer (a) – private	0.71	0.82	0.78	0.85
Healthcare – private	0.40	0.54	0.65	0.59
IT – private	0.55	0.72	0.72	0.77

Table 8.4: Intra-portfolio correlation – some sectors provide some level of diversification to each other

	1 year	3 year	5 year	10 year
1 sector	1.00	1.00	1.00	1.00
2 sectors	0.87	0.78	0.74	0.75
3 sectors	0.82	0.71	0.66	0.67
4 sectors	0.80	0.68	0.61	0.62

Table 8.5: Data-series have a positive skew and some Kurtosis – private

No of sectors	Semi-dev (%)	Skewness	Kurtosis	Sharpe	Sortino
1	12.52	1.44	6.15	0.58	1.35
2	7.76	1.02	4.55	0.81	2.19
3	5.81	0.83	4.03	0.99	2.91
4	4.66	0.72	3.77	1.15	3.63

Table 8.6: Data-series have a positive skew and some Kurtosis – public

No of sectors	Semi-dev (%)	Skewness	Kurtosis	Sharpe	Sortino
1	12.39	0.20	3.61	0.40	0.19
2	8.48	0.15	3.29	0.57	0.28
3	6.77	0.11	3.21	0.70	0.34
4	5.75	0.08	3.14	0.81	0.41

across many GPs). This section will estimate the PETs across the three previously articulated categories (i.e., return-related, risk-related and portfolio level).

In estimating the PETs, instead of looking at the profiles of individual fund of funds, this section uses underlying GP return information as a proxy for constructing/evaluating fund of funds. For this illustration, the dataset included the individual final fund net IRRs from 1991 through 2007 vintages. The dataset was sourced from Preqin in July 2008 and organised into the following fund categories:

- *Buyout:* 433 data points for US and 613 data points for global
- *Secondaries:* 39 data points for US and 46 data points for global
- *Expansion/venture:* 599 data points for US and 820 data points for global
- *All components:* 1071 data points for US and 1479 data points for global

It is possible to use different datasets and time-periods to organise this data into smaller or larger categories as well as to include other categories.
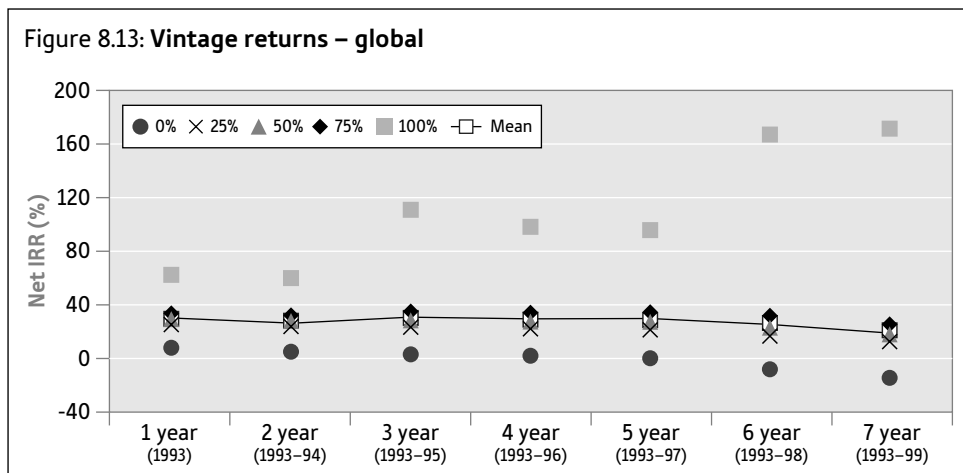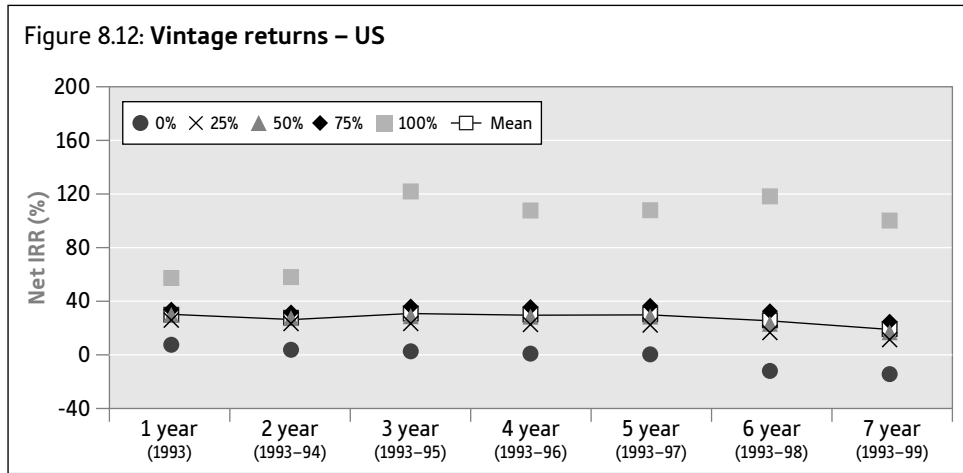
Figure 8.8: **Average quartile IRR depending on number of funds – US buyout**

Figure 8.9: **Average quartile IRR depending on number of funds – global buyout**
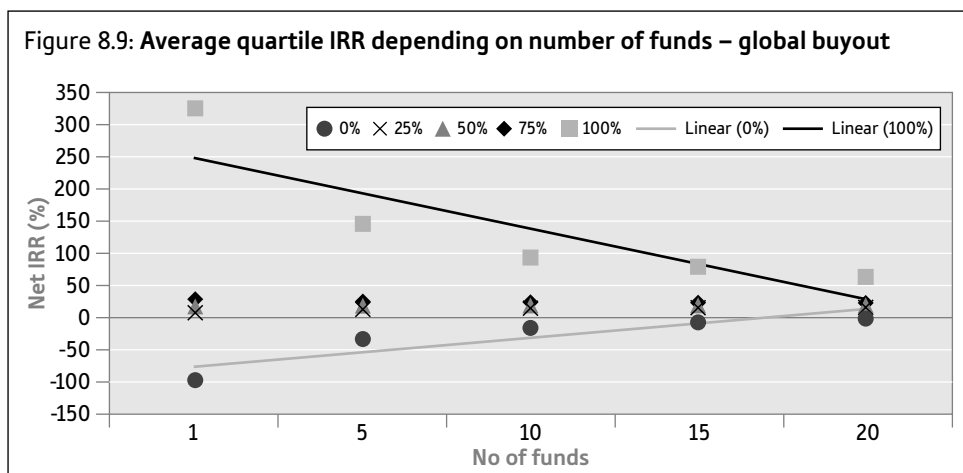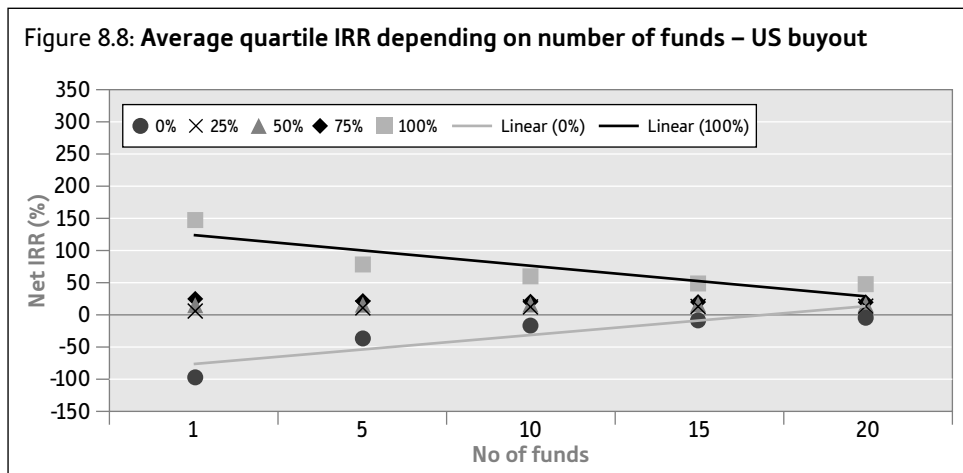
Return-related

Returns have a high degree of dispersion and vary across vintages, categories and number of fund allocations. Figures 8.8 and 8.9 show that the dispersion of returns is greatly reduced as the number of funds is increased (as long as the funds do not have a high degree of positive correlation). Figures 8.10 and 8.11 show that the dispersion within the categories is even more pronounced with expansion/venture exhibiting the highest level of dispersion. Note that Figure 8.11 illustrates the peak and bottom bias highlighted as a shortfall of the Matlab tool, where the peak of the all dispersion is not bounded by the sub-categories. Irrespective, it illustrates the high dispersion levels.

Figures 8.12 and 8.13 show that fund vintage plays a major role in portfolio performance. Although the investment periods of funds provide a high degree of flexibility

Figure 8.10: **Investment type returns – US**

Figure 8.11: **Investment type returns – global**

Figure 8.12: Vintage returns – US

Figure 8.13: Vintage returns – global
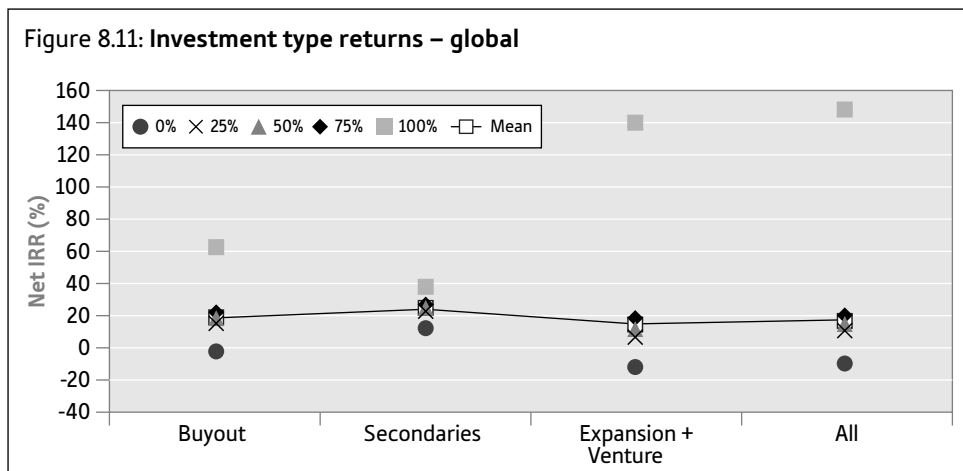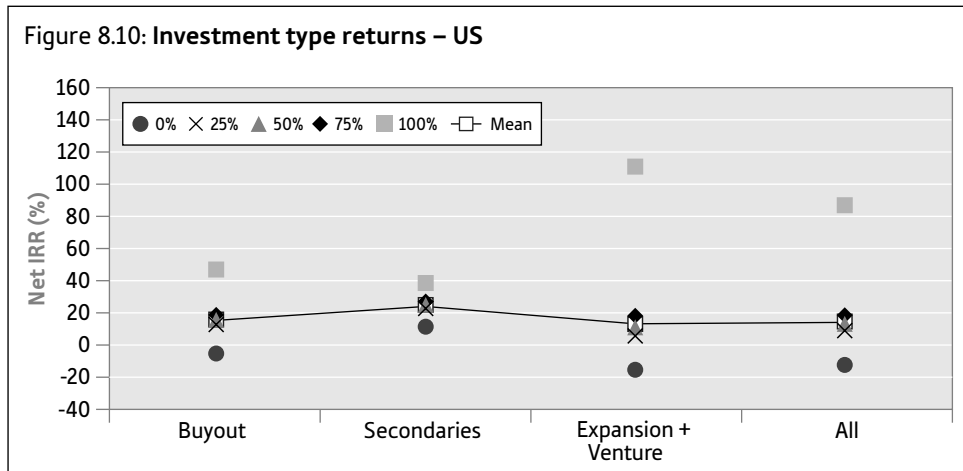
to allocate capital over larger time-horizons, the same vintage funds will generally have some biases including macroeconomic conditions, credit conditions and opportunity set. Conversely, as illustrated in three-years-plus, allocating capital across vintages increases the odds of capturing vintage performance biases. Figure 8.14 shows that the expected return distribution takes on a more familiar or defined profile as more funds are added.

Risk-related

Standard deviation reduces as more funds are included in the portfolio. As illustrated in Figures 8.15 and 8.16, standard deviation falls but beyond approximately 20 funds the marginal impact becomes less pronounced. Given that most funds have between ten and 20 companies within their portfolio, at a 20-fund level a fund of funds is expected to be diversified across 200 to 400 companies. Optimal

Figure 8.14: **Expected return distribution – 2001 vintage year**
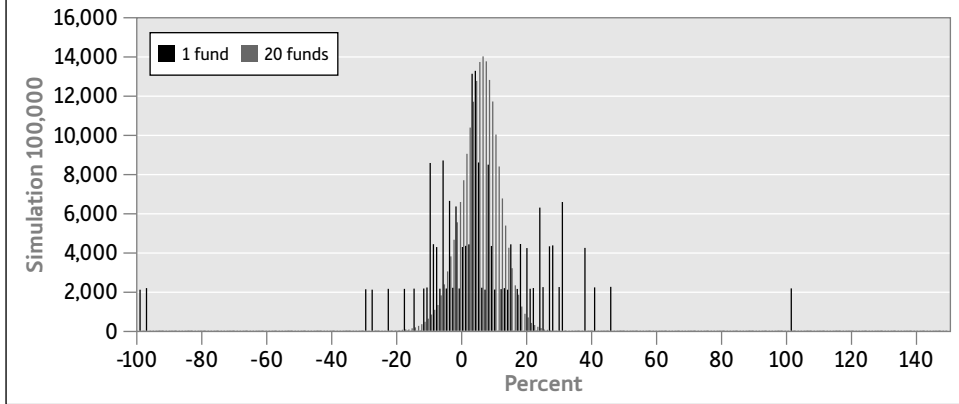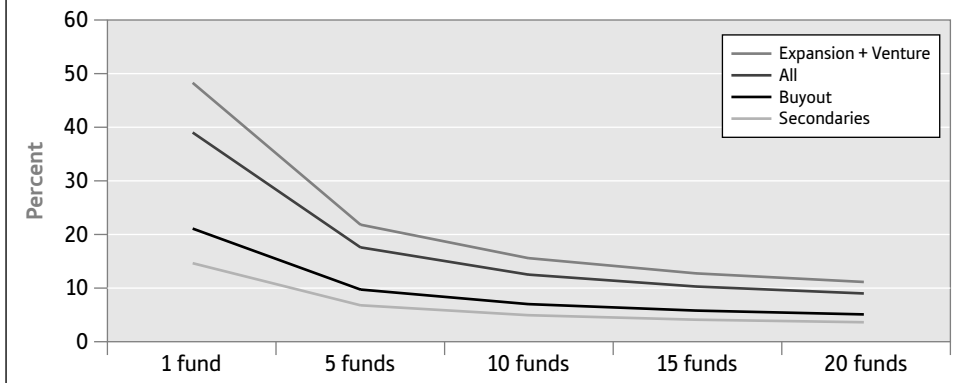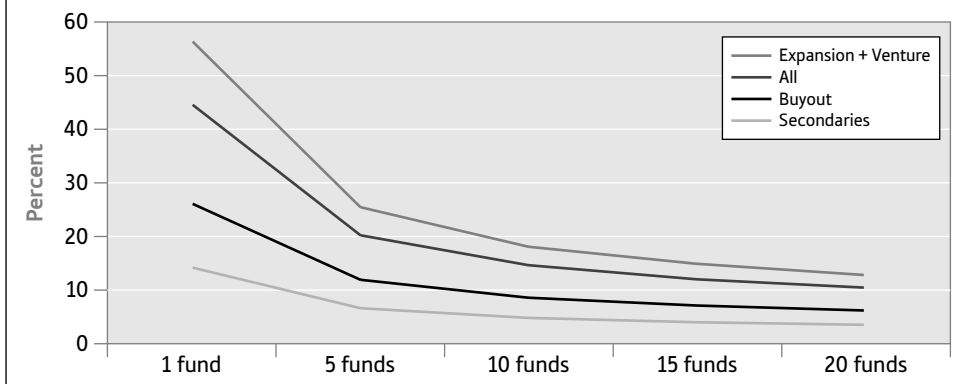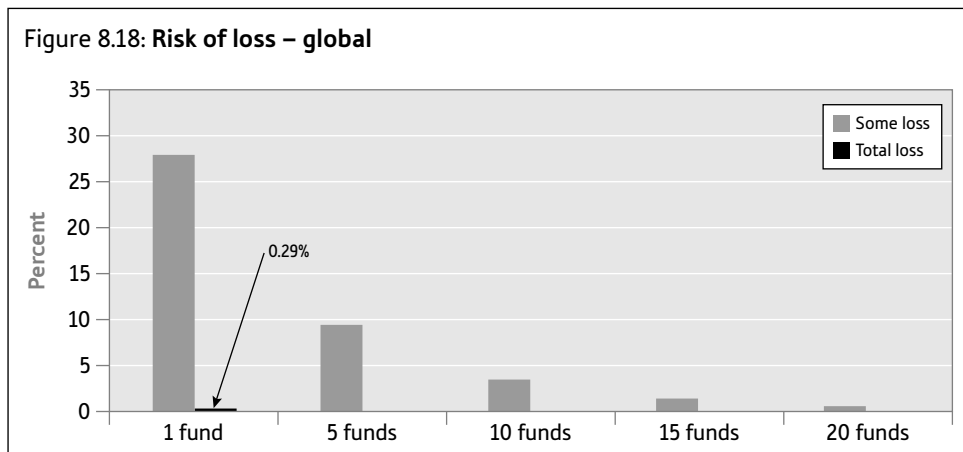

Figure 8.15: **Standard deviation – US**

Figure 8.16: **Standard deviation – global**
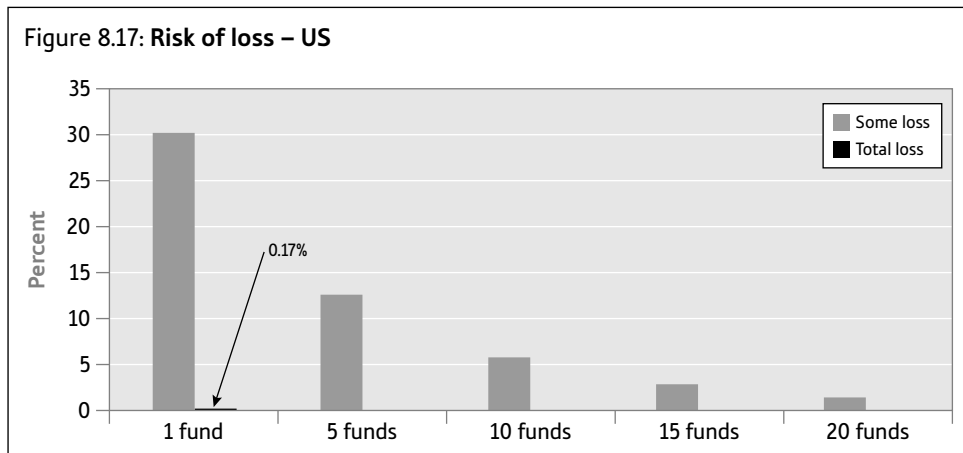
Figure 8.17: **Risk of loss – US**

Figure 8.18: **Risk of loss – global**

management would focus on ensuring that the fund of funds composition has built in diversification across the fund allocations so as not to over diversify or conversely be mislead into believing in the diversification because of sheer numbers (e.g., total allocation to generalist that in the end have a high sector bias due to the prevailing opportunity set). Further, making too many fund allocations always poses the risk of returning the expected return of the portfolio to the mean expected return of the market, which would be substantially under the top-tier return.

Figures 8.17 and 8.18 show that the risk of loss falls as more funds are included. This is in line with the estimates provided in Figures 8.15 and 8.16 and as such also highlights the relative higher risk profile of the GP versus a fund of funds investment.

Portfolio level

Other metrics also vary across categories, return periods as well as vintages. Table

Table 8.7: **Data-series have a positive skew and some categories have more pronounced level of Kurtosis**

No of funds	Semi-dev(%)	Skewness	Kurtosis	Sharpe	Sortino
Buyout – US					
1	9.60	1.27	12.25	0.58	0.74
5	3.03	0.53	4.77	1.29	2.34
10	1.52	0.39	3.93	1.83	NM
15	0.91	0.32	3.57	2.24	NM
20	0.62	0.29	3.50	2.59	NM
Secondaries – US					
1	2.33	0.08	2.03	1.48	NM
5	0.12	0.03	2.81	3.32	NM
10	0.01	0.02	2.90	4.68	NM
15	0.00	0.03	2.96	5.76	NM
20	0.00	0.01	2.94	6.62	NM
Expansion + Venture US					
1	15.85	5.31	45.13	0.19	0.27
5	7.98	2.34	11.07	0.43	0.54
10	5.72	1.68	7.24	0.60	0.74
15	4.56	1.35	5.66	0.75	0.94
20	3.84	1.18	5.08	0.86	1.12
All – US					
1	13.40	5.80	60.64	0.28	0.44
5	5.51	2.58	14.73	0.63	1.07
10	3.49	1.81	8.61	0.89	1.68
15	2.59	1.46	6.67	1.09	2.27
20	2.04	1.27	5.80	1.26	2.88

8.7 shows that the data-series have a positive skew and some categories a more pronounced level of Kurtosis. Further, the Sharpe and Sortino ratios become better as more fund and categories are added.
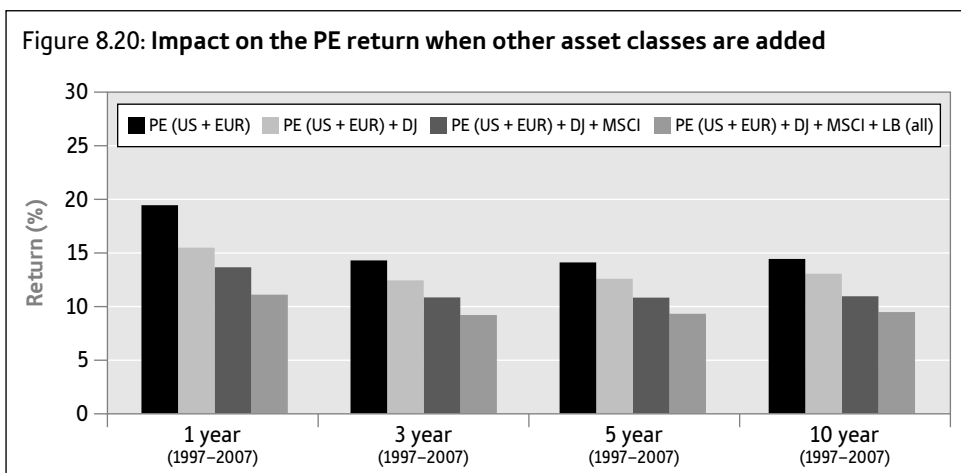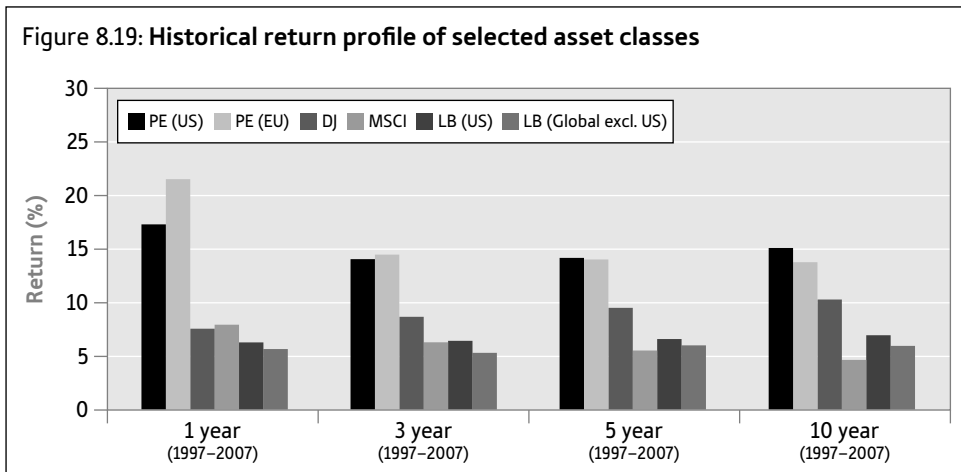
Investor portfolio perspective

This section examines PETs for investors that are evaluating diversifying across the traditional more liquid asset classes (e.g., public equity, bonds etc.) into PE. This section estimates the PETs pre-/post-inclusion of PE within a portfolio from the three previously articulated categories (i.e., return-related, risk-related and portfolio

level). The basic PETs and therein their limitations afforded to the GPs and fund of funds remain the same as to the other investors.

For this analysis, the one-year, three-year, five-year, and ten-year return data was sourced from:

• Private equity returns of pooled industry net IRR from Thomson Financial which includes the information housed by Thomson on all private equity transactions (e.g., buyouts, mezzanine, and venture capital) as sourced in July 2008.
• MSCI World Index returns exclude the US market and is based on data from MSCI Barra.
• Dow Jones Industrial Average returns are based on data from Dow Jones Indexes, a unit of Dow Jones & Company.

Figure 8.19: **Historical return profile of selected asset classes**

Figure 8.20: **Impact on the PE return when other asset classes are added**
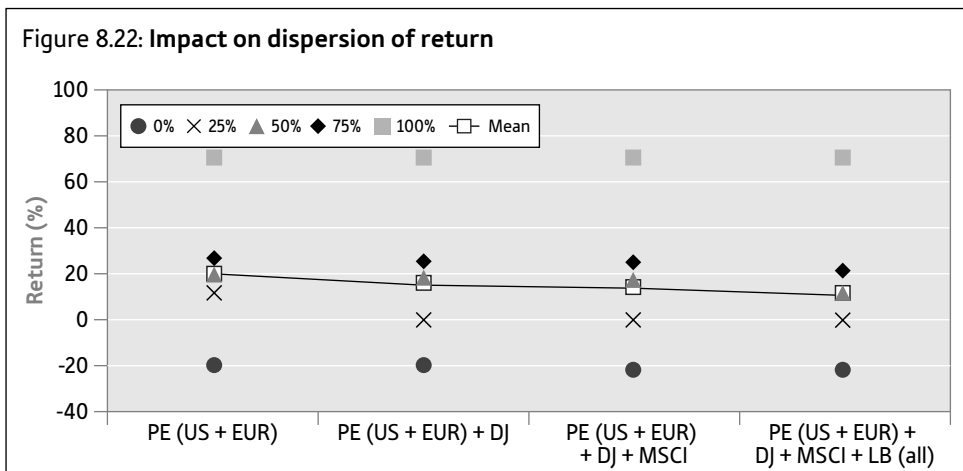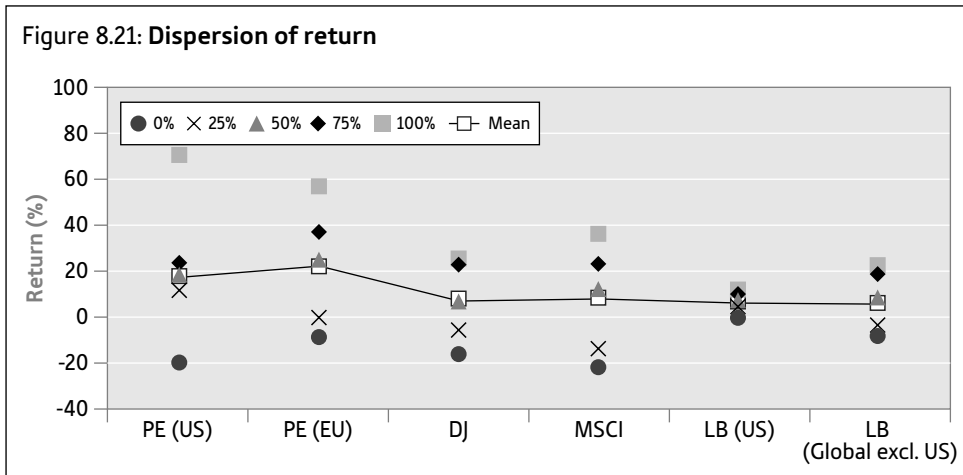
- Lehman Brothers Index returns are for US and World excluding US and are based on data from Lehman Brothers. The three-year, five-year, and ten-year returns were extrapolated from the one-year returns provided by Lehman Brothers.

It is possible to use different datasets, proxies and time-periods to organise this data into smaller or larger datasets as well as to include other asset classes or indices.

Return-related

Returns vary over time and economic cycles. Figure 8.19 shows the historical return profile of the selected asset classes with PE generally over performing the other classes. Figure 8.20 shows the impact on the PE return when other asset classes are included in the portfolio.

Figure 8.21: **Dispersion of return**

Figure 8.22: **Impact on dispersion of return**

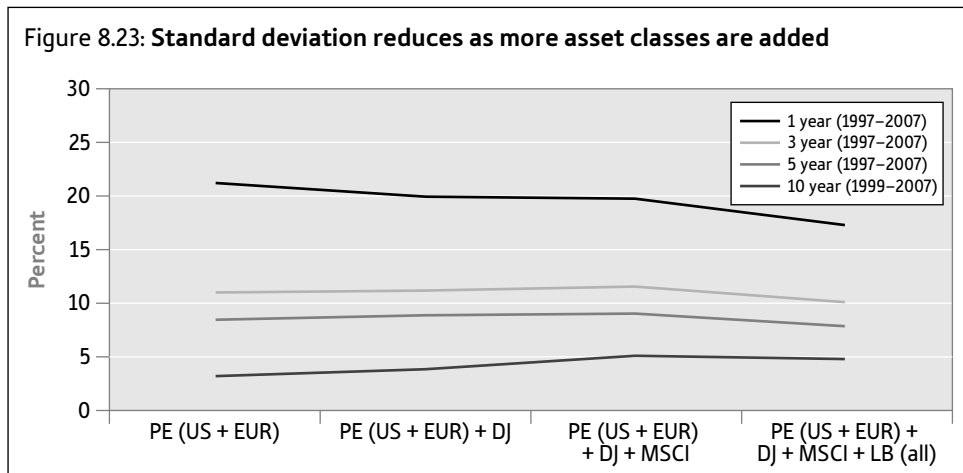**Figure 8.23: Standard deviation reduces as more asset classes are added**

Figure 8.21 shows the dispersion of return, where the bond markets are more tightly distributed followed by the public equity markets and then the private equity markets. Figure 8.22 shows the impact on dispersion of return as more asset classes are included in the portfolio.

Risk-related

Standard deviation generally reduces as more asset classes are added, as shown in Figure 8.23. Some care must be taken in accounting for the PE standard deviations since the pooled IRR series may not be updated very frequently.

Portfolio level

Correlation varies between asset classes and period of returns, as shown in Table 8.8 As expected, PE markets are more closely correlated with the public equity markets than bond markets. An additional item to evaluate further would be that the pooled IRRs used to estimate the PE market returns are generally event-based realisations, which would tend to correlate with the public markets.

Other metrics vary across asset classes and return periods. The analysis captures the basic portfolio management essence of comingling assets that tend to behave differently. Table 8.9 shows that when included, PE tends to have a positive impact on the portfolio.

Conclusion

With a cautionary look towards the theoretical purists, this chapter stated upfront that the multitude of data issues, qualitative overlay and other non-fungible holdings aspects make PET estimation challenging. Data analysis from three different

Table 8.8: **Correlation varies between asset classes and period of returns**

	PE (US)	PE (EU)	DJ	MSCI	LB US	LB (global excl. US)
PE (US)	1	0.8071	0.7270	0.6812	-0.6855	-0.3572
PE (EU)	0.8071	1	0.504	0.4775	-0.5568	-0.5350
DJ	0.7270	0.5040	1	0.8002	-0.4972	-0.0495
MSCI	0.6812	0.4775	0.8002	1	-0.7443	0.1963
LB US	-0.6855	-0.5568	-0.4972	-0.7443	1	0.2374
LB (global excl. US)	-0.3572	-0.5350	-0.0495	0.1963	0.2374	1

Table 8.9: **When included, PE tends to have a positive impact on the portfolio**

	Semi-dev (%)	Skewness	Kurtosis	Sharpe	Sortino
PE (US)	10.52	0.60	4.19	0.66	0.91
PE (EU)	7.02	0.04	2.12	0.94	1.97
DJ	10.20	-0.20	1.77	0.34	-0.03
MSCI	13.92	-0.35	1.91	0.28	0.00
LB (US)	3.71	-0.38	2.19	0.93	-0.44
LB (global excl. US)	9.44	0.06	1.42	0.24	-0.24
PE (US + EUR)	8.95	0.32	3.23	0.79	1.31
PE (US + EUR) + DJ	9.40	0.49	3.52	0.64	0.82
PE (US + EUR) + DJ + MSCI	10.66	0.33	3.44	0.55	0.55
PE (US + EUR) + DJ + MSCI + LB (all)	9.65	0.65	4.36	0.48	0.34

perspectives reveals that in spite of the aforementioned shortfalls the results generally tend to be in line with a PE market practitioner expectations.

- *Return-related:* have a high degree of dispersion and vary across holdings (e.g., sectors, funds, vintages, etc.). Top-quartile performers significantly outperform mean returns.
- *Risk-related:* diversification across lesser correlated assets tends to reduce portfolio volatility as well as risk of loss.
- *Portfolio level:* PE investments generally tend to be the return enhancer and on a risk-adjusted basis inclusion of PE in the portfolio seems to imply greater value.

Further, for GPs, funds of funds and investors there are some noteworthy participant-based observations from the PET analysis.

For GPs:
- Year-on-year sector performance is disparate and impacted by macro-events, credit conditions, market sentiment, prevailing opportunity set, etc. Diversification across sectors generally increases balanced risk-return possibilities.

For fund of funds:
- Fund return quartiles are very disparate but dedicated programmes (e.g., in-house or third-party) with clear strategies should be able to push the portfolio to top-tier returns.
- Diversification across approximately 20 managers balances the risk-return profile and also significantly reduces the risk of loss. However, the programme should account for the company-level diversification attained by the underlying GP investments to ensure that the fund is not overly diversified to drive returns to the sector or PE market's mean return.
- It may not be possible to time the PE market performance but generally, multi-year allocations, diversification across investment types and backing historic alpha generators all increase the odds of top-tier performance.

For investors:
- Inclusion of PE with other asset classes should diversify the portfolio and increase the potential for higher returns.

In conclusion, although a blind, highly quantitative approach to PE management may not be most appropriate, general application of the PETs provides some useful insights. Hopefully this chapter introduced readers to some base PET applications as well as sample illustrations as a means to present the results. The illustrated PETs, datasets, analysis, assumptions should be used as building blocks by readers to create various permutations. □

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[1] Depending on trading or maturity strategies the portfolios may be with or without composition churn during the holding period.

[2] For the purposes of this chapter the focus is on the industry sub-sector as a whole rather unique opportunities within the sub-sector.

[3] It should be pointed out that some of the shortfalls of using the Matlab package include::

- *Random number bias*: In Matlab, the computing is deterministic based on the machine code unless linked to external devices like a gamma ray counter. So the random numbers generated during the simulation may have deterministic sequences. Although each number is expected to be uniformly distributed, the blocks containing 20 contingent numbers from this sequence maybe biased.
- *Peak and bottom bias:* in the quartile analysis, the 0 percent and 100 percent quartile may not be stable as the simulation cannot generate all the possible results. For example, in the fund of funds analysis, for a 20-fund portfolio randomly selected from a 1,000-fund pool, theoretically yields $1,000^{20} = 10^{60}$ possible portfolios. Given the [200,000] iteration simulation the bottom and peak may show biases in such large pool sizes.

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